1 to 9. But 1 multiplied by 10 produces 10, and 9 multiplied by 10 produces 90. And ſo on for all the intermediate units.

X. *A Recommendation of* Hadley's *Quadrant for ſurveying, eſpecially the ſurveying of Harbours, together with a particular Application of it in ſome Caſes of Pilotage. By the Rev.* John Michell, *B. D. F. R. S.*

Read Feb. 14, 1765.

THE uſe of Hadley's quadrant, as an inſtrument to take altitudes at ſea, is already ſo well eſtabliſhed, that it wants no farther recommendation; but there are ſeveral other purpoſes, to which it may be applied, with great advantage, which, though obvious enough, ſeem yet to be hardly ſufficiently attended to. There is no inſtrument ſo well adapted to many kinds of ſurveying, either for exactneſs or conveniency, and particularly the laſt; but the ſurveying of harbours, or ſuch ſands, as lie within ſight of land, may oftentimes be performed by it, not only with vaſtly more eaſe, but alſo with a much greater degree of preciſion, than can be hoped for by any other means, as it is the only inſtrument in uſe, in which neither the exactneſs of the obſervations, nor the eaſe with which they may be taken, are ſenſibly affected by the motion of a veſſel: and hence a ſingle obſerver, in a boat, may generally determine the ſituation of any place he

he pleaſes, with a ſufficient degree of accuracy, if, with this inſtrument, he takes the angles ſubtended by two or three pairs of objects properly choſen upon the ſhores round about him; but it will be ſtill better to have two obſervers, one of whom, being in a boat, muſt, at the time he takes the angle ſubtended by ſome two objects upon the ſhore, make a ſignal to the other obſerver, who, being placed at one of the objects, as a ſtation, muſt at the ſame time obſerve the angle ſubtended by the boat and the other object. By this means, two angles in a plain triangle being given, together with the diſtance between the two objects, as a baſe, the whole triangle, and the ſituation of every part of it, will be given likewiſe. By ſuch obſervations, as theſe, provided the boat be at reſt during the time of making them, and they be made carefully, with good quadrants, though without the aſſiſtance of teleſcopic ſights, the ſituation of places may be eaſily determined to twenty or thirty feet upon every three or four miles.

Beſides the uſe of Hadley's quadrant in ſurveying, it may upon ſome occaſions be very advantageouſly employed in piloting ſhips into harbours, the great readineſs, with which it may be uſed, making it a very convenient inſtrument for this purpoſe: but that this may be done to the greateſt advantage, it will be neceſſary to have a proper proviſion made for it upon the charts, by expreſſing upon them the angles ſubtended by given objects, by means of which, together with the bearings, a ſhip may be enabled to know her ſituation with great exactneſs. The well-known property of the circle, that angles in the ſame ſegment are equal to each other, may be oftentimes

times very conveniently applied upon this occaſion; for if, through any two given objects, we deſcribe ſeveral ſegments of circles, in which thoſe objects ſhall ſubtend the angles of * 120°, 90°, 80°, 70°, &c. reſpectively, we ſhall then know immediately, upon finding the two objects ſubtend any one of theſe angles, that we are ſituated ſomewhere in the circumference of the correſponding ſegment; and, the bearing alſo from one of the objects being known, our preciſe ſituation will be determined with great accuracy.

The manner of deſcribing theſe ſegments, through two given points, may be performed in the following manner. Let B and C [TAB. II. fig. 1.] be the given points: then, joining theſe two points, biſect the line B C in A, and draw the indefinite right line DE, perpendicular to B C, through the point of biſection. Upon BC, at the point C conſtitute the angles D C B, F C B, G C B, H C B, &c. reſpectively equal to the difference between the angles, which correſpond to the ſeveral intended ſegments and 90°; and on the oppoſite or ſame ſide of the line C B with thoſe ſegments, accordingly as they exceed or fall ſhort of 90°. Then will the points D, F, G, H, &c. where the lines C D, C F, C G, C H, &c. interſect the line D E, be the centres of

* The number and frequency of theſe ſegments, as well as the magnitude of their reſpective angles, muſt be determined, according to the particular circumſtances of the occaſion, upon which they are applied: I have mentioned no greater angle, than 120°, as there are few caſes, in which this will not be ſufficient; and indeed it is the greateſt, that Hadley's quadrant, the only inſtrument fit for this purpoſe, will eaſily admit, according to the preſent conſtruction of it.

Fig. 1.

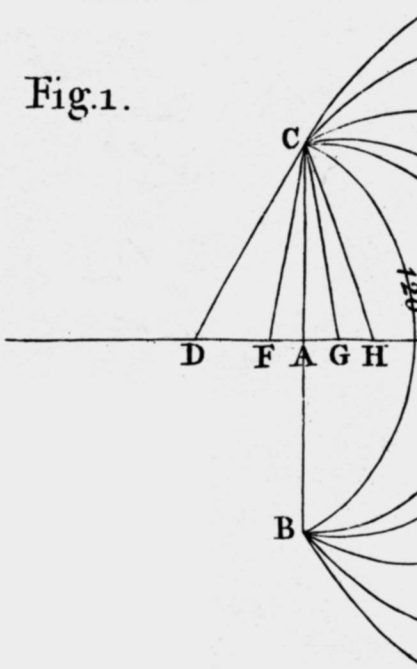


Fig. 2.

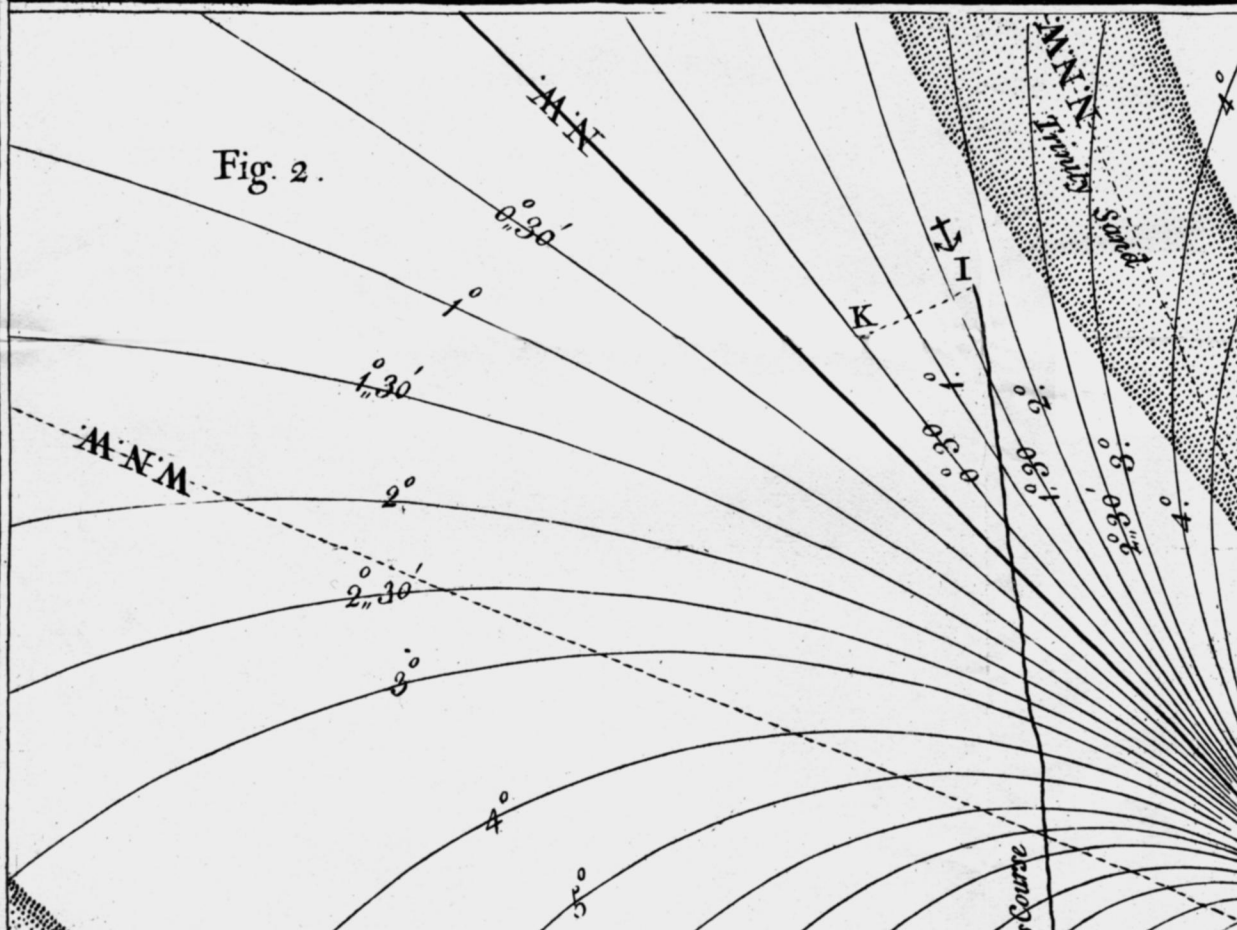
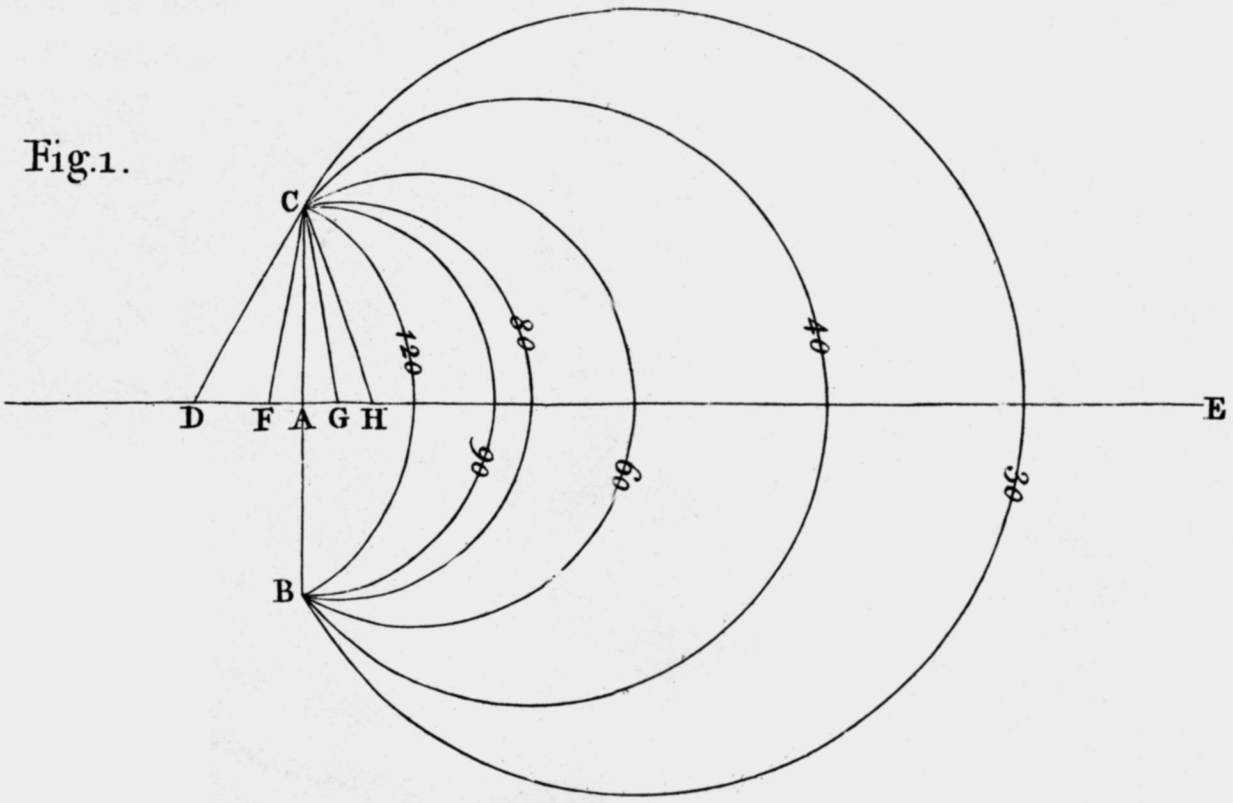
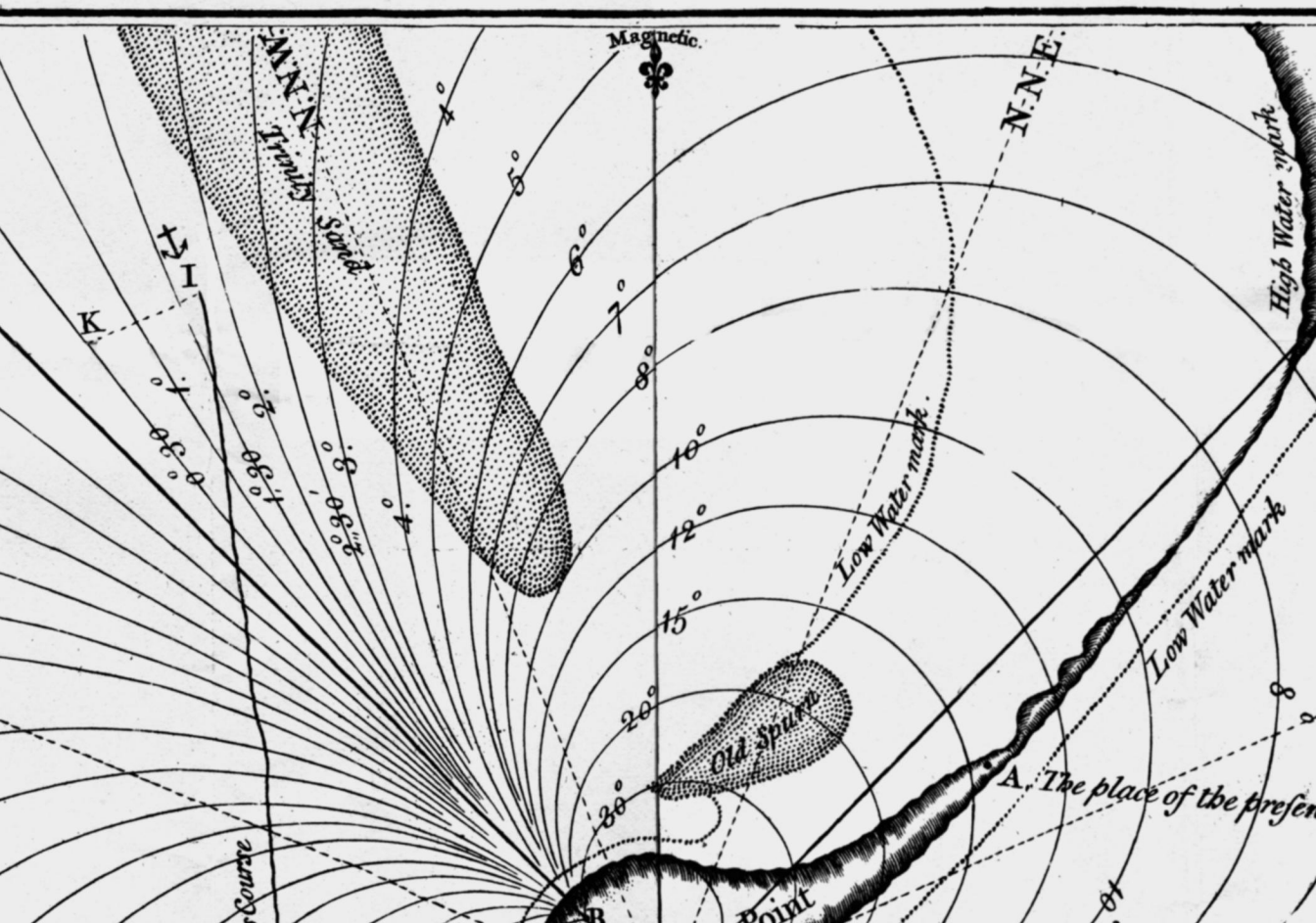


Fig.1.


C
D
F
A
G
H
E
B
120
80
40
100
60
20


Magnetic.
N.N.E.
N.N.W.
Tide's Set
High Water mark
Low Water mark
Low Water mark
Old Spurn
A. The place of the
Point
K
I
4°
5°
6°
7°
8°
10°
12°
15°
20°
30°
0.30
1.00
2.00
2.30
4°

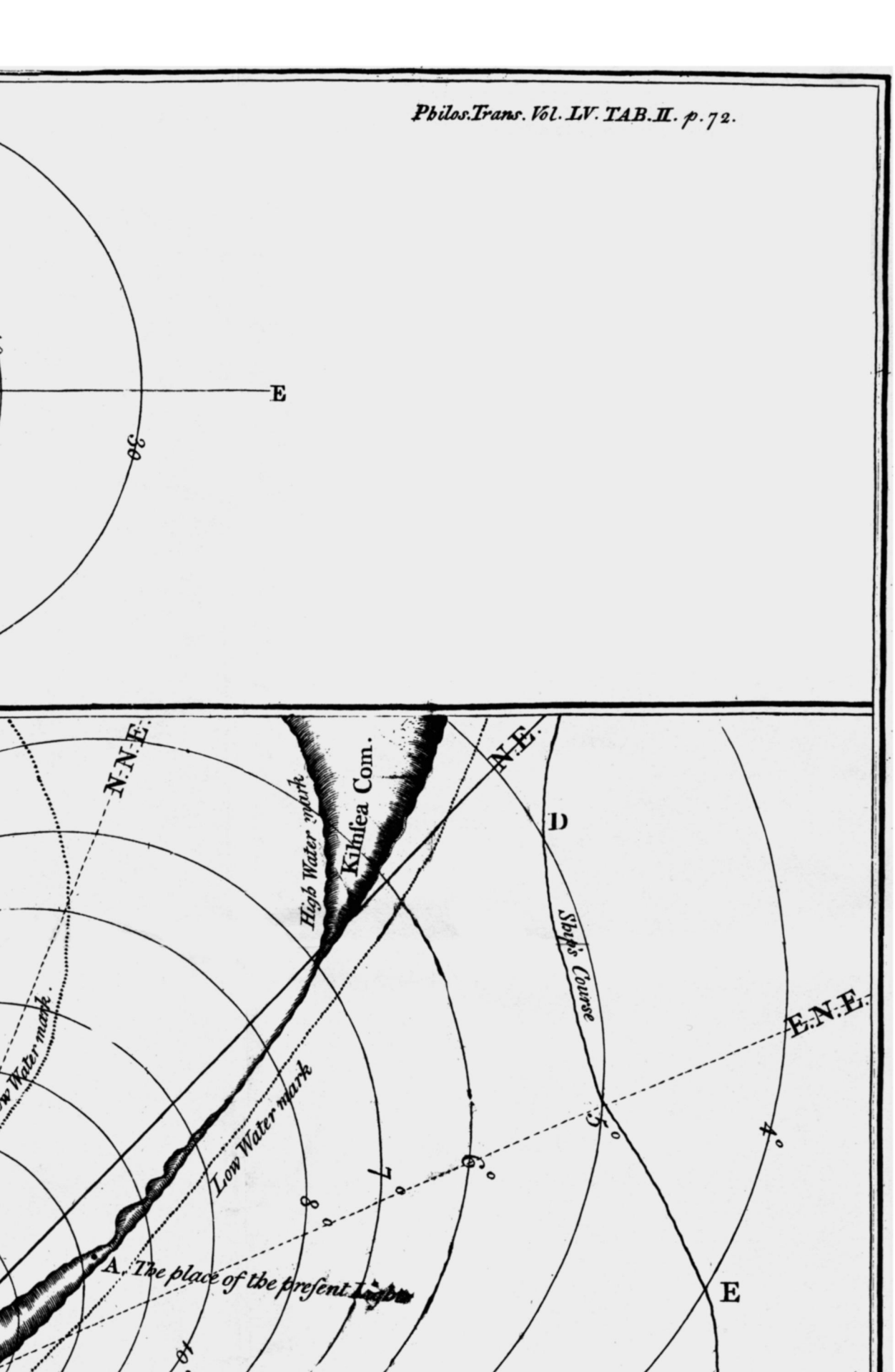
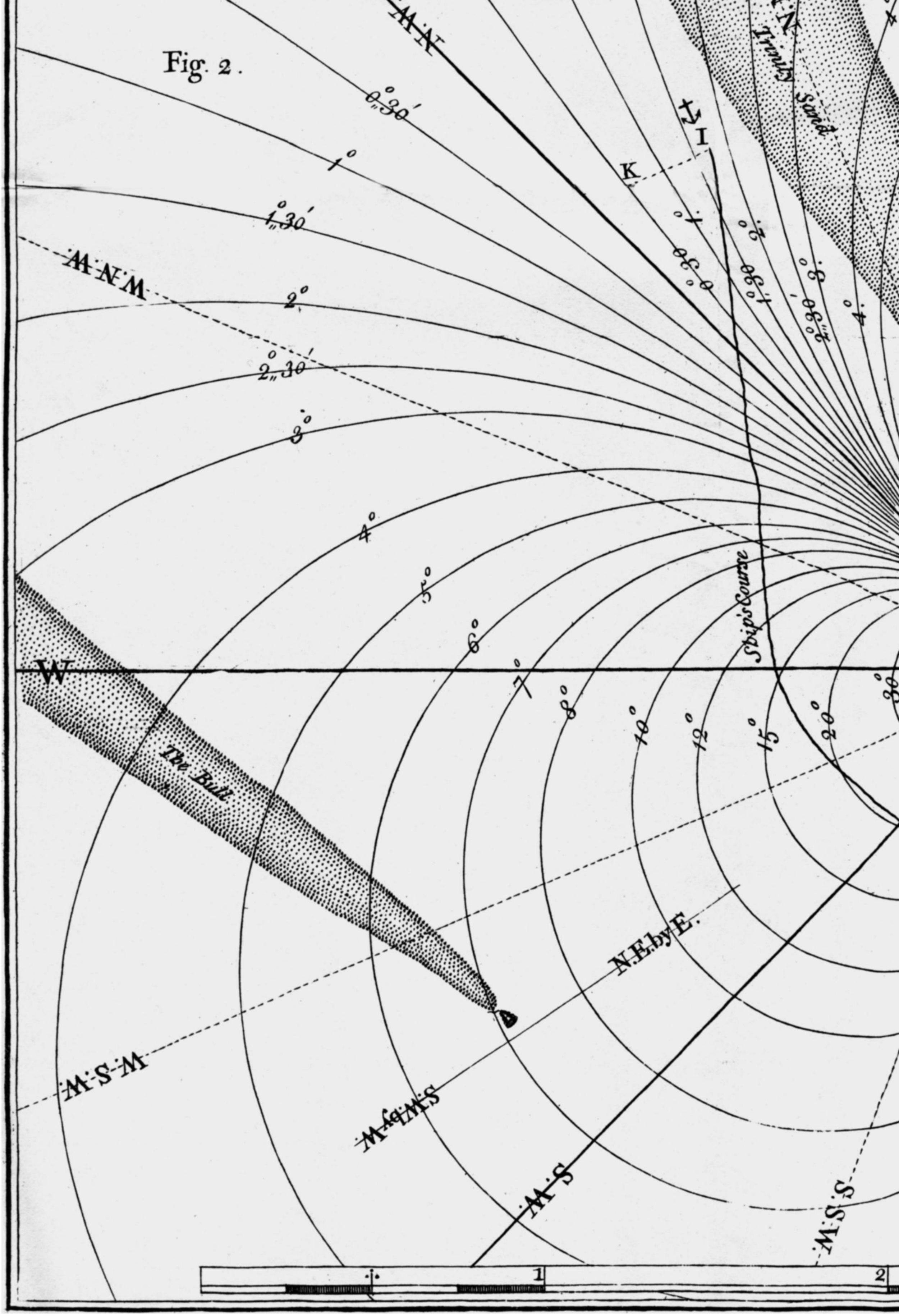
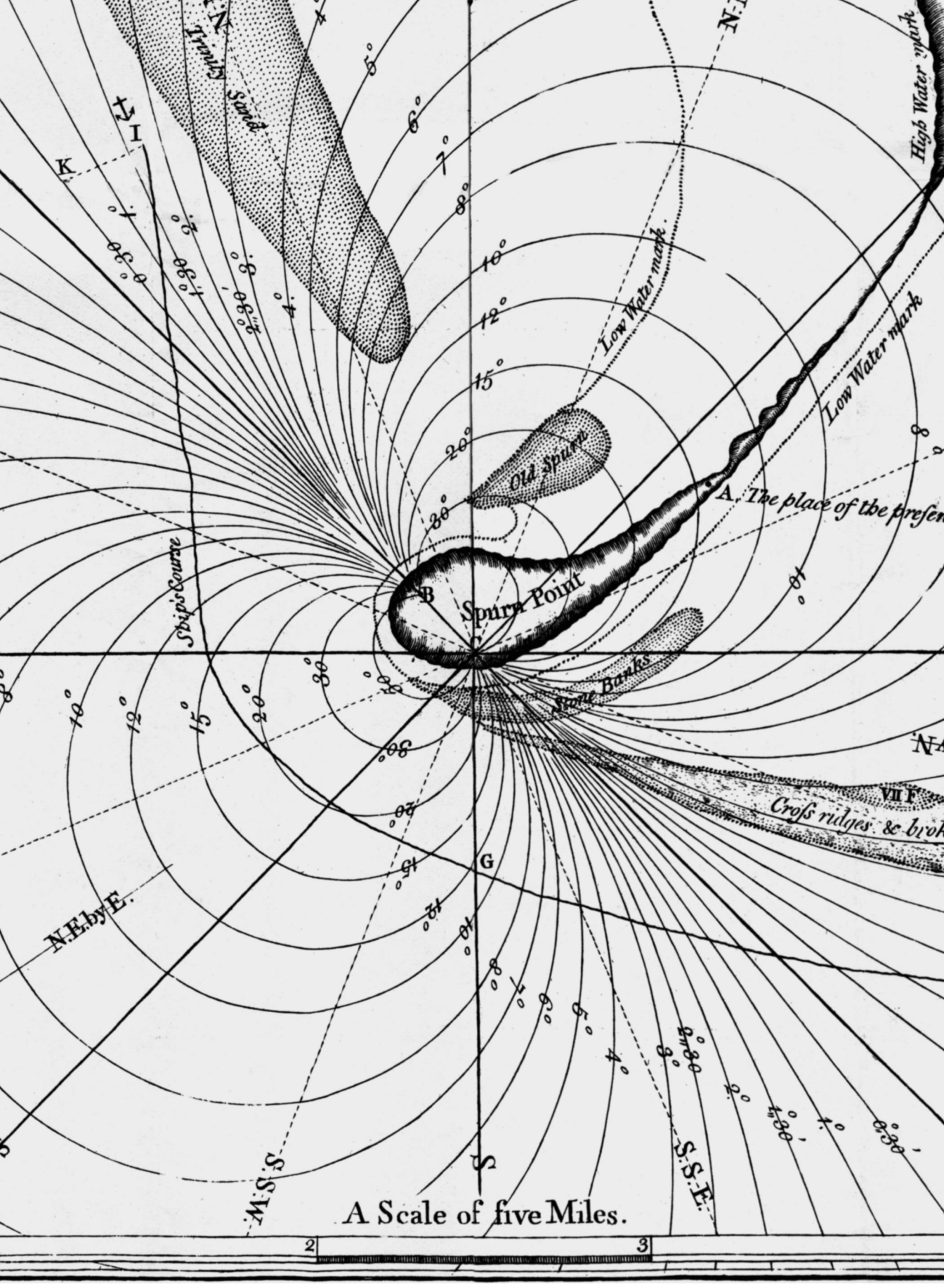
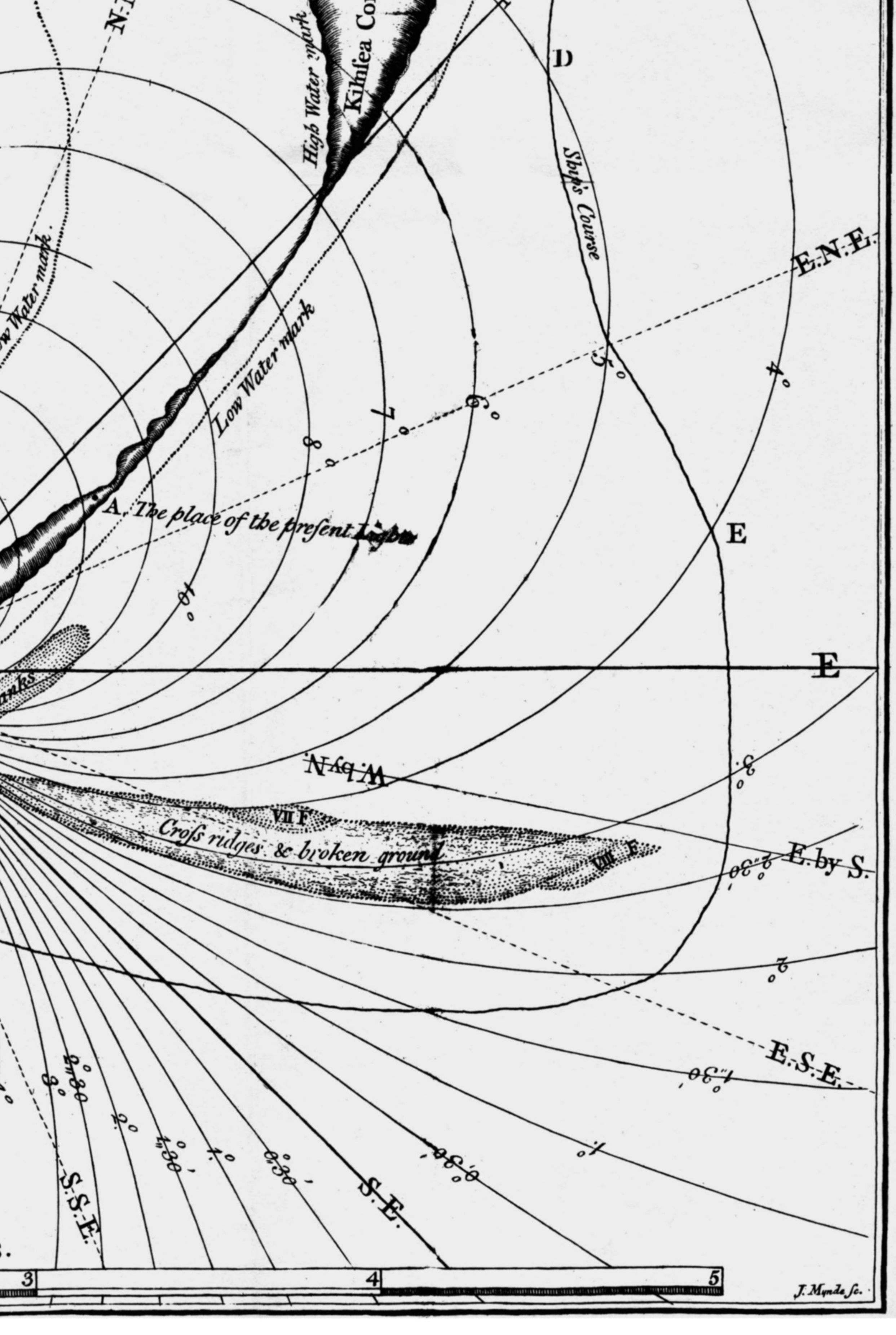

E
N.N.E.
High Water mark
Kilnſea Com.
N.E.
D
Ship's Course
E.N.E.
Low Water mark
A. The place of the preſent Light
E

Fig. 2.



Trinity Sand
I
K
W.N.W.
W
The Bull
N.E.by E.
W.S.W.
S.W.by W.
S.W.
S.S.W.
Ships Course
0°30′
1°
1°30′
2°
2°30′
3°
4°
5°
6°
7°
8°
10°
12°
15°
20°
1
2


Sand
Low Water mark
Low Water mark
High Water mark
Old Spurn
A The place of the preser
Spurn Point
B
Stone Banks
Cross ridges & brok
Ship's Course
K
I
G
N.E.by E.
S.S.W.
S.
S.S.E.
A Scale of five Miles.
2
3



High Water mark
Kilnsea
D
Ship's Course
E.N.E.
Low Water mark
Low Water mark
A. The place of the present Light
E
E
W.by N.
VII F
Cross ridges & broken ground
VIII F
E. by S.
E.S.E.
S.S.E.
S.E.
3
4
5
J. Mynde sc.

the intended ſegments; but, if the angle, correſponding to the intended ſegment, is to be neither greater or leſs than 90°, the point A, which biſects the line B C, will be the centre of the intended ſegment.

Thus, if I would have the angle in the intended ſegment, to conſiſt of 120°, I conſtitute upon B C, at the point C, the angle D C B equal to 30°, the difference between 120° and 90°, and on the oppoſite ſide of C B from its correſponding ſegment, becauſe it exceeds 90°. Then with the centre D, and radius D C, I deſcribe the ſegment C 120 B, in every part of which the two points C and B will ſubtend an angle of 120°.

In like manner, if I would have the angle in the intended ſegment to conſiſt of 80°, I conſtitute upon B C, at the point C, the angle B C G equal to 10°, the difference between 80° and 90°, and becauſe the intended angle is now leſs than 90°, I place G on the ſame ſide of B C with its correſponding ſegment. Then with the centre G, and radius G C, I deſcribe the ſegment C 80 B, in every part of which the two points C and B will ſubtend an angle of 80°.

Laſtly, if I would have the angle, in the intended ſegment, to conſiſt of 90°, with the centre A and radius A C, I deſcribe the ſemicircle C 90 B, in every part of which the two points C and B will ſubtend an angle of 90°*.

* If we want a ſegment, the angle in which is to be the half of one we have got before, the centre is already found to our hands; for it will be the point of interſection of the former ſegment with the perpendicular line D E. Thus the point,

The demonſtration of all this is ſo well known to every geometrician, that it needs not to be inſerted; but it may not perhaps be amiſs to exemplify what I mean, by applying to it a particular caſe.

The preſent inconvenient and indeed dangerous ſituation of the two lights at the mouth of the Humber, commonly called the Spurn Lights, muſt probably ſoon make it neceſſary to remove them; for the ground, upon which they formerly ſtood, is now ſo far waſhed away, as not to leave ſufficient room to erect them at a proper diſtance from each other; and freſh ground being grown up to the ſouthward, ſo as to make the point above a mile diſtant from them, ſhips are frequently thereby liable to be deceived. In caſe therefore theſe lights ſhould, at any time hereafter, be removed nearer to the point, I think the foregoing principle might be very conveniently and advantageouſly applied, ſo as to enable entire ſtrangers to enter the Humber, with the greateſt

where the ſegment, correſponding to 120°, interſects the line D E, will be the centre of the ſegment correſponding to 60°: the point, where the ſegment correſponding to 80°, interſects the line D E, will be the centre of the ſegment correſponding to 40°, &c.

There is another method of finding the centres of theſe ſegments, which, if great exactneſs is required, will perhaps be found preferable, in practice, to the former, eſpecially in the larger ſegments. The method, I mean, is to ſet them off by a ſcale, in which caſe the diſtance of the ſeveral centres, from the point A, muſt be reſpectively proportional to the co-tangents of the angles in their correſponding ſegments, or, what is the ſame thing, reſpectively proportional to the tangents of the angles D C A, F C A, G C A, H C A, &c. conſtituted upon the line A C, at the point C; for, making A C the radius, the lines D A, F A, G A, H A, &c. are the reſpective tangents of thoſe ſeveral angles.

ſecurity,

ſecurity, even in the darkeſt nights, provided only they could ſee the lights: and this is the more material, as, in dangerous weather, it is frequently the beſt and moſt ſecure retreat between the Thames and the Tyne; but the difficulty of entering it, for want of proper helps, oftentimes obliges ſhips to keep the ſea, not without great difficulty and hazard, when they might lie here in perfect ſafety.

I have endeavoured to give ſome idea of the application, I propoſe, in the annexed chart, [Fig. 2.] which is meant only as a rough draught of the general deſign, and not as an accurate delineation of what muſt depend upon a future ſurvey, whenever the new lights ſhall be erected.

The ſtrong black line is intended to repreſent the high water mark: the ſpace included within this, and which is inſcribed Spurn Point, as well as that inſcribed Kilnſea Common, is ground, which the ſea never overflows; but all the ſpace between theſe, where the ſingle line only is continued, is frequently waſhed over by the ſea, the land being there reduced, at the time of high water, to a meer bank of only a few yards wide.

The dotted line ſurrounding the ſtrong black line repreſents the low water mark; and the other ſpaces included within dotted lines, and inſcribed Trinity ſand, the bull, croſs ridges and broken ground, and ſtone banks, are ſuppoſed to repreſent four ſands, the two former lying within, and the two latter without the mouth of the Humber.

Through the two points B and C, repreſenting the ſuppoſed new lights, are deſcribed, according to the rule above laid down, two ſeries of ſegments, one

ſeries to the ſouth weſt, and the other to the north eaſt of them; but, the north eaſtern ſeries lying towards the land ſide, a few of the inner ſegments might be omitted as being unneceſſary.

From the point C are drawn lines in the direction of ſixteen of the points of the compaſs; every other point being omitted in order to make the chart more diſtinct; for no greater nicety is in general wanted, than what theſe will afford; near the extremities however of two of the ſands, where it may perhaps be more neceſſary, I have added a ſhort line to mark the intermediate point.

I ſhall now endeavour to illuſtrate the whole by an imaginary ſhip's courſe. Let us ſuppoſe then a ſhip to enter the ſpace, repreſented upon the chart, from the north, and finding the light C to bear nearly ſouth weſt, and the two lights to ſubtend an angle of about 5°, ſhe will know herſelf to be ſomewhere about the place D, a little leſs eaſternly, than the point of the ſand denominated croſs ridges and broken ground, and at the diſtance of about two miles and a quarter from it, as may appear by the ſcale; ſhe muſt therefore ſteer ſomewhat to the eaſtward of the ſouth, and, having run about three quarters of a mile, ſhe will find the light C to bear weſt ſouth weſt, and the two lights to ſubtend an angle of exactly 5°, being now in the interſection of the courſe, delineated upon the chart, with the ſegment ſo marked; but not being yet far enough to the eaſtward, ſhe muſt ſtill continue the ſame direction, till, having run near three quarters of a mile farther, ſhe will find the two lights ſubtending an angle of no more than 4°, and the light C lying a little to the ſouthward of the weſt. From hence

hence ſhe may know herſelf to be at the place E, ſomewhat more to the eaſtward, than the ſand ſhe wants to avoid, and at the diſtance of almoſt a mile from it. She may now therefore ſafely change her courſe, and ſtand due ſouth, till having brought the light C to bear weſt by north ſhe is right over-againſt the extremity of the ſand, at the diſtance of about three or four hundred yards. If ſhe ſtill keeps the ſame courſe about three hundred yards farther, the lights will then ſubtend an angle of only 2° 30′, when ſhe will be ſo far to the ſouth, as to be clear of all danger, though ſhe ſhould from thenceforward ſteer due weſt; but, for greater ſecurity, we will ſuppoſe her ſtill continuing to ſtretch away a little more to the ſouthward, by ſteering ſouth weſt for about a quarter of a mile farther. From hence then ſhe may keep near due weſt, with which courſe the two lights, in a run of a mile and a quarter, or a little more, will both be brought to bear north weſt, and be ſeen together. In a run of about half a mile more, they will be again open on the contrary ſide from the former to about 3°, from which they will open to 10°, in a paſſage of near half a mile farther, when the ſhip will be arrived at the place G, the light C bearing then due north. The change of the angle being now very great in a very ſmall diſtance, it will be very eaſy to keep the ſhip with great exactneſs to whatever part of the channel we pleaſe. Let us however ſuppoſe, that we mean to keep her in the courſe laid down in the chart; from G then we muſt ſteer north weſt a little weſt, and in a run of about a mile, we ſhall firſt increaſe the angle of the two lights, till it is ſomewhat more than 20°, the light C then bearing

ing north eaſt, and afterwards diminiſh it again to about 13°, when the light C will bear due eaſt. Having now got far enough within the Spurn, we may run due north about a mile and half, firſt bringing the two lights to bear in one line due ſouth eaſt, and then opening them again, on the oppoſite ſide, to an angle of about 1° 30′ or 2°, according to our diſtance within the Spurn *; but here, knowing perfectly well where we are, we can come to an anchor in a ſafe road, at ſuch a diſtance from the Trinity ſand as we pleaſe, which diſtance may be regulated without difficulty by the ſoundings, as at the place I, where the anchorage is very good in five fathom water.

* In the caſe of a ſhip's being placed nearly in the line of the two lights, the method of finding our diſtance from them by their bearing, and the angle they ſubtend, will fail us; but, as it may ſometimes be convenient to have the means of forming a tolerable gueſs about our exact ſituation in this caſe, as well as others, I muſt obſerve that a pretty good judgment may be made concerning it, by running a little way in a direction nearly at right angles to the line of the lights; for by this means, the diſtance run being known, it will appear, from the change of the angle, under which they are ſeen, how far they are from us. Thus, for example, if from the place I, a veſſel ſhould run about a quarter of a mile along the dotted line I K, and find, that this made a change in the angle ſubtended by the two lights of a very little more than a degree, ſhe would know herſelf to be almoſt two miles from the light C, as would appear by meaſuring the diſtance I C upon the ſcale.